SHAPE IDENTIFICATION OF FORCED HEAT-CONVECTION FIELDS

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ABSTRACT

This paper presents a numerical analysis method for solving shape identification problem of temperature distribution prescribed problem in sub-domains of steady heat convective fields. Let Ω be a heat convective fields in a steady state. The heat fluid flows in from sub-boundaries Γ₀ and flows out from sub-boundaries Γ₁, where we write velocity vector u = {uᵢ}ᵢ=₁ⁿ, pressure p, temperature θ. A domain variation problem where the temperature distribution θ is specified with θ₀ in sub-domains Γ₀ ⊂ Ω can be regarded as a shape optimization problem. For simplicity, we assume that the sub-domains Γ₀ and Γ₁ are invariables. This problem is formulated as

Find \( \Omega \) that minimizes

\[
E(\theta) = E(\theta - \theta_D, \theta - \theta_D) = \int_{\Omega_D} (\theta - \theta_D)^2 \, dx
\]

subject to

\[
a^V(u, w) + b(u, w) + c(w, p) = l(w) \quad \forall w \in W
\]
\[
c(u, q) = 0 \quad \forall q \in Q
\]
\[
a^H(\theta, \xi) + d(u, \theta, \xi) + h^H(\theta, \xi) = f_q(\xi) + f_h(\xi) \quad \forall \xi \in \Xi
\]
\[
\int_{\Omega} dx \leq M
\]

where Eqs.(3), (4) and (5) are variational forms, or weak forms, using adjoint velocity \( w = \{w_i\}^n_{i=1} \), adjoint pressure \( q \) and adjoint temperature \( \xi \) for the state equations. Eq.(6) is the constraint with respect to the volume. The terms such as the \( a^V(u, w) \) are defined as

\[
a^V(u, w) = \frac{1}{Re} \int_\Omega w_{i,j} (u_{i,j} + u_{j,i}) \, dx, \quad b(v, u, w) = \int_\Omega w_{ij} u_{i,j} \, dx, \quad c(w, p) = -\int_\Omega w_i p \, dx,
\]
\[
l(w) = \int_{\Gamma_i} w_i \sigma_i d\Gamma, \quad a^H(\theta, \xi) = \frac{1}{Pe} \int_\Omega \theta_{j,k} \xi_{j,k} \, dx, \quad d(u, \theta, \xi) = \int_\Omega \xi u_j \theta_{j} \, dx,
\]
\[
h^H(\theta, \xi) = \int_{\Gamma_h} \theta \hat{h} d\Gamma, \quad f_q(\xi) = \int_{\Gamma_q} \hat{q} d\Gamma, \quad f_h(\xi) = \int_{\Gamma_h} \hat{h} \theta_{j} d\Gamma
\]

where Reynolds number \( Re \), Peclet number \( Pe \), the traction \( \sigma_i \), the heat flux \( \hat{q} \), the heat transfer coefficient \( \hat{h} \) and the ambient temperature \( \theta_j \) are given as known values or functions.

Applying the concept of the Lagrange multiplier method and the adjoint variable method, this problem can be rendered as a stationary problem for the Lagrange functional \( L(u, p, \theta, w, q, \xi, \Lambda) \):

\[
L = E(\theta - \theta_D, \theta - \theta_D) - a^V(u, w) - b(u, w) - c(w, p) + l(w) - c(u, q)
\]
\[
- a^H(\theta, \xi) - d(u, \theta, \xi) - h^H(\theta, \xi) + f_q(\xi) + f_h(\xi) + \Lambda \int_\Omega dx - M
\]
where \( \Lambda \) is the Lagrange multiplier with respect to the volume constraint. The derivative \( \dot{L} \) with respect to domain variation for shape optimization is calculated. Letting this \( \dot{L} = 0 \), the Kuhn-Tucker conditions with respect to \( u, p, \theta, w, q, \xi, \Lambda \) are obtained by

\[
\begin{align*}
\frac{\partial V}{\partial u'}(u, w') + b(u, u', w) + c(w, p) = l(u') & \quad \forall u' \in W \\
c(u', q') = 0 & \quad \forall q' \in Q \\
a_H(\theta, \xi') + d(u, \theta, \xi') + h_H(\theta, \xi') &= f(\xi') + f_h(\xi') & \quad \forall \xi' \in \Xi \\
a_V(\xi', w) + b(u', u, w) + b(u, u', w) + c(u', q) + d(u', \theta, \xi) = 0 & \quad \forall u' \in W \\
c(u, p') = 0 & \quad \forall p' \in Q \\
a_H(\theta', \xi) + d(u, \theta', \xi) + h_H(\theta', \xi) &= 2E(\theta - \theta_D, \theta') & \quad \forall \theta' \in \Theta \\
\Lambda \geq 0, & \quad \int_{\Omega} dx \leq M, \quad \Lambda(\int_{\Omega} dx - M) = 0
\end{align*}
\]

that indicate the variational forms of the original state equations for \( u, p \) and \( \theta \), the variational forms of the adjoint equations for \( w, q \) and \( \xi \) which we call adjoint equations, respectively. Where \( (\cdot)' \) is the shape derivative for domain variation of the distributed function fixed in spatial coordinates. Under the condition satisfying Eqs. (8)- (14), the derivative \( \dot{L} \) agrees with the linear form \( <G\nu, V> \) with respect to the velocity function \( V \) of domain variation:

\[
\dot{L}|_{u,p,\theta,w,q,\xi,\Lambda} = <G\nu, V> = \int_{\Gamma} G\nu V_i d\Gamma,
\]

\[
G = G_0 + G_1 A,
\]

\[
G_0 = -\frac{1}{Re} \nabla_i u_i + u_i u_i - \frac{1}{Pr} \nabla_k \theta_k \nabla \cdot (\hat{h} \theta \xi) - (\hat{h} \theta \xi) \kappa + \nabla \cdot (\hat{h} \bar{\xi} \xi) + (\hat{h} \bar{\xi} \xi) \kappa,
\]

\[
G_1 = 1
\]

where \( \nu \) is an outward unit normal vector on the boundary, \( \nabla \cdot (\cdot) \equiv \nabla (\cdot) \cdot \nu \) and \( \kappa \) denotes the mean curvature.

The coefficient vector function \( G\nu \) in Eq. (15) has the meaning of a sensitivity function relative to domain variation and is so-called the shape gradient function. The scalar function \( G \) is called the shape gradient density function. Since the shape gradient function is obtained, the traction method [1][2] can be applied to this shape optimization problem.

The successful numerical results of 2D branch channel problem, where the temperature distribution \( \theta \) is specified with \( \theta_D = 40 \) in sub-domain \( \Omega_D \), shows the validity of the present method in Fig. 1.

**REFERENCES**
