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SHAPE IDENTIFICATION OF FORCED HEAT-CONVECTION FIELDS

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ABSTRACT

This paper presents a numerical analysis method for solving shape identification problem of temperature distribution prescribed problem in sub-domains of steady heat convective fields.

Let Ω be a heat convective fields in a steady state. The heat fluid flows in from sub-boundaries Γ_0 and flows out from sub-boundaries Γ_1, where we write velocity vector $u = \{u_i\}_{i=1}^n$, pressure p, temperature θ. A domain variation problem where the temperature distribution θ is specified with θ_D in sub-domains $\Omega_D \subset \Omega$ can be regarded as a shape optimization problem. For simplicity, we assume that the sub-domains Ω_D, sub-boundaries Γ_0 and Γ_1 are invariables. This problem is formulated as

$$\text{Find} \quad \Omega \tag{1}$$

$$\text{that minimizes} \quad E(\theta) = E(\theta - \theta_D,\ \theta - \theta_D) = \int_{\Omega_D} (\theta - \theta_D)^2\, dx \tag{2}$$

$$\text{subject to} \quad a^V(u,w) + b(u,u,w) + c(w,p) = l(w) \qquad \forall w \in W \tag{3}$$

$$c(u,q) = 0 \quad \forall q \in Q \tag{4}$$

$$a^H(\theta,\xi) + d(u,\theta,\xi) + h^H(\theta,\xi) = f_q(\xi) + f_h(\xi) \qquad \forall \xi \in \Xi \tag{5}$$

$$\int_{\Omega} dx \le M \tag{6}$$

where Eqs.(3), (4) and (5) are variational forms, or weak forms, using adjoint velocity $w = \{w_i\}_{i=1}^n$, adjoint pressure q and adjoint temperature ξ for the state equations. Eq.(6) is the constraint with respect to the volume. The terms such as the $a^V(u,w)$ are defined as

$$a^V(u,w) = \frac{1}{Re}\int_{\Omega} w_{i,j}(u_{i,j} + u_{j,i})\,\mathrm{d}x, \quad b(v,u,w) = \int_{\Omega} w_i v_j u_{i,j}\,\mathrm{d}x, \quad c(w,p) = -\int_{\Omega} w_{i,i} p\,\mathrm{d}x,$$

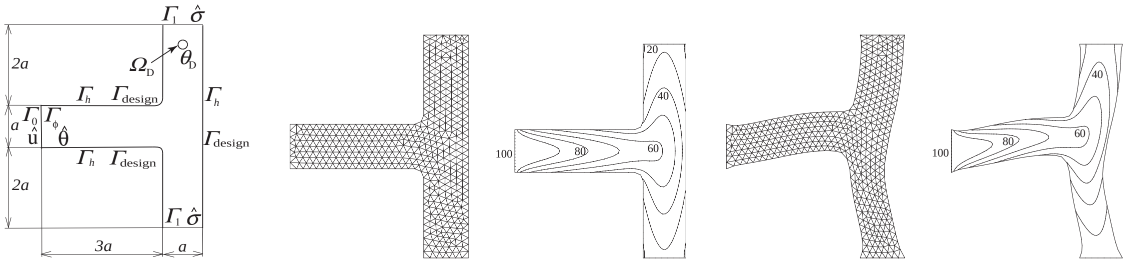
$$l(w) = \int_{\Gamma_1} w_i \hat{\sigma_i}\,\mathrm{d}\Gamma, \quad a^H(\theta,\xi) = \frac{1}{Pe}\int_{\Omega} \theta_{,k}\xi_{,k}\,\mathrm{d}x, \quad d(u,\theta,\xi) = \int_{\Omega} \xi u_j \theta_{,j}\,\mathrm{d}x,$$

$$h^H(\theta,\xi) = \int_{\Gamma_h} \theta\xi\hat{h}\,\mathrm{d}\Gamma, \quad f_q(\xi) = \int_{\Gamma_q} \xi\hat{q}\,\mathrm{d}\Gamma, \quad f_h(\xi) = \int_{\Gamma_h} \xi\hat{h}\hat{\theta_f}\,\mathrm{d}\Gamma$$

where Reynolds number Re, Peclet number Pe, the traction $\hat{\sigma_i}$, the heat flux $\hat{q}$, the heat transfer coefficient $\hat{h}$ and the ambient temperature $\hat{\theta_f}$ are given as known values or functions.

Applying the concept of the Lagrange multiplier method and the adjoint variable method, this problem can be rendered as a stationary problem for the Lagrange functional $L(u,p,\theta,w,q,\xi,\Lambda)$:

$$\begin{aligned} L = {} & E(\theta - \theta_D,\ \theta - \theta_D) - a^V(u,w) - b(u,u,w) - c(w,p) + l(w) - c(u,q) \\ & - a^H(\theta,\xi) - d(u,\theta,\xi) - h^H(\theta,\xi) + f_q(\xi) + f_h(\xi) + \Lambda\Big(\int_{\Omega} dx - M\Big) \end{aligned} \tag{7}$$



(a) Problem (b) Initial shape (c) Identified shape

Figure 1: Numerical results for 2D branch channel problem, shapes and temperature distributions

where Λ is the Lagrange multiplier with respect to the volume constraint. The derivative $\dot{L}$ with respect to domain variation for shape optimization is calculated. Letting this $\dot{L} = 0$, the Kuhn-Tucker conditions with respect to $u, p, \theta, w, q, \xi, \Lambda$ are obtained by

$$a^V(u, w') + b(u, u, w') + c(w', p) = l(w') \quad \forall w' \in W \tag{8}$$

$$c(u, q') = 0 \quad \forall q' \in Q \tag{9}$$

$$a^H(\theta, \xi') + d(u, \theta, \xi') + h^H(\theta, \xi') = f_q(\xi') + f_h(\xi') \quad \forall \xi' \in \Xi \tag{10}$$

$$a^V(u', w) + b(u', u, w) + b(u, u', w) + c(u', q) + d(u', \theta, \xi) = 0 \quad \forall u' \in W \tag{11}$$

$$c(w, p') = 0 \quad \forall p' \in Q \tag{12}$$

$$a^H(\theta', \xi) + d(u, \theta', \xi) + h^H(\theta', \xi) = 2E(\theta - \theta_D,\ \theta') \quad \forall \theta' \in \Theta \tag{13}$$

$$\Lambda \geq 0, \quad \int_\Omega dx \leq M, \quad \Lambda(\int_\Omega dx - M) = 0 \tag{14}$$

that indicate the variational forms of the original state equations for u, p and θ, the variational forms of the adjoint equations for w, q and ξ which we call adjoint equations, respectively. Where $(\,\cdot\,)'$ is the shape derivative for domain variation of the distributed function fixed in spatial coordinates. Under the condition satisfying Eqs. (8)- (14), the derivative $\dot{L}$ agrees with the linear form $< G\nu, V >$ with respect to the velocity function V of domain variation:

$$\dot{L}|_{u,p,\theta,w,q,\xi,\Lambda} =< G\nu, V >= \int_\Gamma G\nu_i V_i \, d\Gamma, \tag{15}$$

$$\begin{aligned} &G = G_0 + G_1 \Lambda, \\ &G_0 = -\frac{1}{Re} w_{i,j}(u_{i,j} + u_{j,i}) - \frac{1}{Pe}\theta_{,k}\xi_{,k} - \nabla_\nu(\hat{h}\theta\xi) - (\hat{h}\theta\xi)\kappa + \nabla_\nu(\hat{h}\hat{\theta_f}\xi) + (\hat{h}\hat{\theta_f}\xi)\kappa, \\ &G_1 = 1 \end{aligned} \tag{16}$$

where ν is an outward unit normal vector on the boundary, $\nabla_\nu(\,\cdot\,) \equiv \nabla(\,\cdot\,) \cdot \nu$ and κ denotes the mean curvature.

The coefficient vector function $G\nu$ in Eq. (15) has the meaning of a sensitivity function relative to domain variation and is so-called the shape gradient function. The scalar function G is called the shape gradient density function. Since the shape gradient function is obtained, the traction method [1][2] can be applied to this shape optimization problem.

The successful numerical results of 2D branch channel problem, where the temperature distribution θ is specified with $\theta_D = 40$ in sub-domain Ω_D, shows the validity of the present method in Fig. 1.

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