MORPHING AND FITTING TECHNIQUES OF FINITE-ELEMENT MODELS USING THE TRACTION METHOD

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Abstract

This paper presents two types of methods, that we call a morphing method and a fitting method, to deform finite-element models for skeletal parts to accord with medical image data, such as X-ray photographs and voxel image data by the CT scanner or MRI. The morphing method is proposed to pick up nodes on a finite-element model and move them to desired locations under suitable constraints referring to X-ray photographs or CT image data keeping the smoothness of the boundary. The fitting method is proposed to deform a finite-element model to fit into a domain extracted from a voxel image data. These methods are derived as solutions to shape optimization problems using the traction method. Morphing a spine finite-element model to accord with a patient of idiopathic scoliosis and fitting a finite-element model of the third lumbar vertebra to a CT image data were demonstrated.

Key Words : Shape Optimization, Finite-Element Method, Traction Method, Morphing Method, Fitting Method

1 Introduction

Computational mechanics has been applied not only to industry, but also to medical treatment. One of the most difficulties to apply the computational methods, such as the finite-element method, to the human body is how to construct numerical models from medical image data, such as X-ray photographs and voxel image data by the CT scanner or MRI.

Recently, a construction method of finite-element models for skeletal parts, such as the femur and the lumbar vertebra, from CT image data was developed by using a technique of contour extraction from cross section image data and automatic generation techniques of triangular surface patches and tetrahedral mesh.

However, some difficulties remain in generation of hexahedral structured mesh and in applicability for large scale models, such as for the spine system. For buckling analyses to investigate etiology of the idiopathic scoliosis for patients and to analyze effective reinforcement parts for treatment by sensitivity analysis based on the optimization theory, quality of the finite-element meshes of the spine are crucial.

To overcome the difficulties, a feasible way is to deform qualified finite-element models to accord with the medical image data. This paper presents two types of deforming techniques based on shape optimization problems, that we call a morphing method and a fitting method. These methods are derived as solutions to the shape optimization problems using the traction method that was proposed as a solution to the boundary shape optimization problems of domains in which boundary value problems of partial differential equations are defined [1, 2] and that was improved [3].

2 Morphing method

Let us suppose to pick up nodes on a finite-element model and move them to desired locations under suitable constraints referring to X-ray photographs or CT image data. The key to the successful moving is how to keep the smoothness of the
2.1 Morphing problem

Let us consider that a linear elastic continuum defined in a bounded domain $\Omega \subset \mathbb{R}^n$ ($n = 2, 3$), its boundary $\Gamma$, is expected to move to a bounded reference domain $\Omega_{\text{ref}} \subset \mathbb{R}^n$, especially that specified points $x_k \in \Omega$ ($k = 1, 2, \ldots, q$) move to target positions $x_k^{\text{ref}} \in \Omega_{\text{ref}}$ ($k = 1, 2, \ldots, q$) respectively as shown in Fig. 1. This moving problem of domain involving a volume constraint can be formulated as

$$
\min_{\Omega \subset \mathbb{R}^n} \int_\Omega \sum_{k=1}^q \delta(x-x_k) \|x_k^{\text{ref}} - x\|^2 \, dx \quad \text{such that} \quad \int_\Omega dx = \int_{\Omega_{\text{ref}}} dx
$$

where $\delta(\cdot)$ denotes the Dirac delta function.

Shape gradient of this problem can be obtained by the Lagrange multiplier method. Using $\Lambda$ as a Lagrange multiplier for the volume constraint, the Lagrange functional $L$ and its derivative $\dot{L}$ with respect to domain variation is obtained using the formulation of material derivative as

$$
L = \int_\Omega \sum_{k=1}^q \delta(x-x_k) \|x_k^{\text{ref}} - x\|^2 \, dx + \Lambda \left( \int_\Omega dx - \int_{\Omega_{\text{ref}}} dx \right)
$$

$$
\dot{L} = -2 \int_\Omega \sum_{k=1}^q \delta(x-x_k) \left( x_k^{\text{ref}} - x \right) \cdot V \, dx + \Lambda \left( \int_\Gamma v \cdot V \, d\Gamma + \Lambda \left( \int_\Omega dx - \int_{\Omega_{\text{ref}}} dx \right) \right)
$$

where $(\cdot)$, $v$ and $V$ denote the material derivative, the outward unit normal vector and the velocity of the domain variation respectively. In Eq.(3), the relations $\delta(x-x_k) = 0$ and $\dot{x} = V$ were used.

Using $\Lambda$ determined to satisfy the volume constraint, the derivative of Lagrange functional is obtained as

$$
L|_\Lambda = \int_\Omega G_0 \cdot V \, dx + \int_\Gamma G_1 v \cdot V \, d\Gamma \equiv \langle G_0, V \rangle + \langle G_1 v, V \rangle
$$

$$
G_0 \equiv -2 \sum_{k=1}^q \delta(x-x_k) \left( x_k^{\text{ref}} - x \right), \quad G_1 \equiv \Lambda
$$

where $G_0$ and $G_1 v$ are the shape gradients for the objective functional and the volume constraint in this problem respectively. In this paper, the sign $\equiv$ is used for definition.
2.2 Morphing method using the traction method

Let us consider to reshape with the shape gradients $G_0$ and $G_1 \nu$ obtained above. Paying attention to that $G_0$ consists of the Dirac delta functions, lack of smoothness in the shape gradients is worried even if the traction method was applied. Actually, in the case of moving a point on a beam-like finite-element model to the target point shown in Fig. 2(a), a sharp-pointed shape as shown in Fig. 2(b) was obtained by the traction method.

Recalling that the traction method works as a smoother of boundary in reshaping [2], it is expected that to repeat the traction method improves the smoothness of the boundary. Actually, applying the traction method to the result shown in Fig. 2(b), the smooth boundary was obtained as shown in Fig. 2(c). The procedure of the morphing method that we propose is to repeat the improved traction method using the Robin condition [3] as follows.

(i) Solve $V^{[1]} \in D : a(V^{[1]}, y) + \alpha \langle (V^{[1]} \cdot \nu) \nu, y \rangle = -(G_0, y) - \langle G_1 \nu, y \rangle \forall y \in D$.

(ii) Substitute $V^{[1]}$ on $\Gamma$ into $G^{[2]}$ and solve $V^{[2]} \in D : a(V^{[2]}, y) + \alpha \langle (V^{[2]} \cdot \nu) \nu, y \rangle = (G^{[2]}, y) \forall y \in D$.

(iii) Iterate to substitute $V^{[m-1]}$ on $\Gamma$ into $G^{[m]}$ and to solve $V^{[m]} \in D : a(V^{[m]}, y) + \alpha \langle (V^{[m]} \cdot \nu) \nu, y \rangle = (G^{[m]}, y) \forall y \in D$ until satisfied smoothness is obtained.

The bilinear forms $\langle \cdot, \cdot \rangle$ and $(\cdot, \cdot)$ are defined in Eq. (4). The boundary stiffness parameter $\alpha > 0$ is required to set up according to the problems [3]. The bilinear form $a(\cdot, \cdot)$ is defined as

$$a(V, y) \equiv \int_{\Omega} C_{ijkl} V_{i,j} y_{i,j} \, dx$$

(6)

where $\{C_{ijkl}\}_{i,j,k,l=1,2,\ldots,n}$ is the stiffness of the linear elastic continuum. In Eq. (6), the Einstein summation convention and the gradient notation $(\cdot)_i \equiv \partial(\cdot)/\partial x_i$ were used. The set $D$ is defined as

$$D = \{ V \in (H^1(\Omega))^n \mid \text{shape constraints} \}.$$  

(7)

2.3 Numerical Example

One of the most effective objects of the morphing method is to construct a spine finite-element model according with a patient of idiopathic scoliosis. Figure 3(a) shows a X-ray photograph of a patient with which a spine finite-element model is expected to accord. Morphing condition was assumed with moving the point on the surface at the eighth thoracic vertebra under fixing the bottom surface and the top point of the model. Figures 3(b) and (c) illustrate the results of the first try and the second try of the traction method. More precise accordance is going to be obtained by using additional moving conditions.
3 Fitting method

Let us assume that a finite-element model is positioned near the desired location for example by the morphing method and consider that the finite-element model deforms to fit a voxel image data by the CT scanner or MRI.

3.1 Fitting problem

Let us consider that a linear elastic continuum defined in a bounded domain \( \Omega \subset \mathbb{R}^n \) (\( n = 2, 3 \)), its boundary \( \Gamma \), that is included in a domain \( \Omega_{\text{limit}} \subset \mathbb{R}^n \) for a voxel image data is expected to move to a bounded reference domain \( \Omega_{\text{ref}} \subset \Omega_{\text{limit}} \subset \mathbb{R}^n \), its boundary \( \Gamma_{\text{ref}} \), as shown in Fig. 4.

To formulate a fitting problem, let us introduce a signed distance function \( \phi : \Omega_{\text{limit}} \ni x \mapsto \phi \in \mathbb{R} \) defined as

\[
\phi(x) \equiv \chi_{\Omega_{\text{ref}}}(x)d(x), \quad \chi_{\Omega_{\text{ref}}}(x) = \begin{cases} 
-1 & (x \in \Omega_{\text{ref}}) \\
1 & (x \in \Omega_{\text{ref}}^c)
\end{cases}
\]

(8)

where \( d(x) : \Omega_{\text{limit}} \ni x \mapsto d \in \mathbb{R} \) denotes the distance at \( x \in \Omega_{\text{limit}} \) from the boundary \( \Gamma_{\text{ref}} \) as shown in Fig. 4. Using \( \phi(x) \), a fitting problem can be formulated as

\[
\min_{\Omega \subset \mathbb{R}^n} J := \int_{\Omega} \phi(x)dx.
\]

(9)

In this paper, the sign \(:=\) is used as the meaning of “that is”. Actually, it can be confirmed that the objective functional \( J \) becomes minimum when \( \Omega \) accords with \( \Omega_{\text{ref}} \).

Using the formulation of material derivative, the derivative of the objective functional \( J \) with respect to domain variation is obtained as

\[
J = \int_{\Gamma} \phi(x) \mathbf{v} \cdot \mathbf{V}d\Gamma \equiv \langle G\mathbf{v}, \mathbf{V} \rangle, \quad G \equiv \phi(x)
\]

(10)

where \( G\mathbf{v} \) is the shape gradient for the objective functional in this problem.
3.2 Fitting method using the traction method

When the target domain $\Omega^{\text{ref}}$ consist of a voxel image data, it will not be effective to evaluate the signed distance in the strict sense of the definition. Considering that the traction method works as a smoother of boundary in reshaping, it can be consider evaluating the signed distance as constant in each voxel.

The procedure of the fitting method that we propose is to repeat the improved traction method using the shape gradient $G\nu$ of the fitting problem as follows.

(i) Extract contour from each cross section of a voxel image data and set $d = 0$ for the voxels on the contour.

(ii) Allocate a distance unit to $d$ for the voxels contacting with the voxels on the contour and twice distance unit $d$ for the voxels contacting with the voxels with the distance unit, and iterate in a similar fashion for all the voxels.

(iii) Iterate to solve $V \in D : a(V, y) + a((V \cdot \nu)\nu, y) = \langle \chi_{\Omega^{\text{ref}}}(x)d(x), y \rangle \ \forall y \in D$ and to reshape with $\Delta sV$ using an incremental parameter $\Delta s$ until the objective functional is converged.

3.3 Numerical Example

A finite-element model of the third lumber vertebra was fitted into a CT image data by the fitting method as shown in Fig. 5. In this analysis, domain of the finite-element model consisting of cortical bone was chosen as $\Omega$. Therefore, the finite-elements consisting of cancellous bone were followed by the deformation of the finite-elements of cortical bone.

4 Conclusion

This paper has presented two types of methods called by the morphing method and the fitting method.

The morphing method was proposed as a solution of a shape optimization problem to minimize the squared distance between the specified nodes of the finite-element model and the target positions. Although the shape gradient of the
problem consists of the Dirac delta functions that have lacking in smoothness, the repeating use of the traction method recovered the insufficiency.

The fitting method is proposed as a solution of a shape optimization problem to minimize the integral of the signed distance that was defined as the distance from the boundary of the target domain with the positive and the negative signs for outside and inside of the target domain respectively. The shape gradient of the problem consists of the signed distance. Considering that the target domain consist of a voxel image data and that the traction method works as a smoother of boundary in reshaping, to evaluate the signed distance as constant in each voxel and to reshape by the traction method were proposed.

To confirm the validity of the two methods, morphing a spine finite-element model to accord with a patient of idiopathic scoliosis and fitting a finite-element model of the third lumbar vertebra to a CT image data were demonstrated.

References

