An Approach to Solving Shape Optimization Problems and Its Application to Mechanical Design

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Abstract

Shape optimization problems of linear elastic continua, flow fields, magnetic fields, etc. under equilibrium and eigenvalue conditions can be generalized as an optimization problem of domains in which elliptic boundary value problems are defined. The optimization theory for these problems was formulated by expressing domain variation with a mapping defined in an initial domain. Using this theory, a derivative of an objective functional with respect to domain variation can be derived rigorously. It is known, however, that ordinary domain optimization problems lack sufficient regularity. This paper describes a new technique for overcoming this irregularity and its application to actual problems.
Key Words: Optimum Design, Shape Optimization, Material Derivative Method, Gradient Method, Traction Method, Finite Element Method

1 Introduction

In classical mechanics, equilibrium and eigenvalue conditions of linear elastic continua, flow fields, magnetic fields, etc. are given by elliptic differential equations defined in terms of domains and boundary conditions. When design variables are considered as the geometrical shapes of the domains, the problems are called geometrical domain or shape optimization problems.

The theoretical basis concerning derivation of sensitivity functions, which we call shape gradient functions, to geometrical domain variation has been studied from early in this century. In 1908, Hadamard[1] showed the differentiability of variations of a geometrical domain with a smooth boundary in which an elliptic boundary value problem is defined.[2] Zolésio[3] extended the theory to domains with piecewise smooth boundaries. He formulated domain variation with a smooth transformation, or mapping, of Euclidean space just the same as the original domain. He called the domain variation the velocity field and this approach the material derivative method. The applicability of the material derivative method to engineering problems was demonstrated by Haug, Choi and Komkov.[4]

Although the shape gradient functions can be evaluated rigorously, the essential difficulty of domain optimization problems is the lack of regularity. The theoretical basis for the necessary regularity can be found in Chenais's work.[5] He showed that if a domain has a boundary which is Lipschitz continuous, a domain optimization problem has a solution. A numerical analysis showing the necessity of the Lipschit condition can be found in a monograph published by Haslinger and Neittaanmäki.[6] Braibant and Fleury[7] presented numerical results that indicated unrealistic shapes were generated by moving nodes in a finite element mesh. Based on that observation, they proposed the use of B-spline curves to control shapes. Kikuchi, Chung, Torigaki and Taylor[8] showed that the optimal shapes strongly depends on the shapes of finite elements near the design boundary. As a remedy, they proposed the application of adaptive finite element methods.

Following the works of these pioneers, one of the authors proposed a regularization technique which we call the traction method.[9] This technique is based on the idea of a gradient method in Hilbert space, which was shown by Cea.[10] Starting with the linear form with respect to the velocity, Cea demonstraited the use of a coercive bilinear form in Hilbert space to determine the velocity that minimizes the objective functional. Our proposal is to use the bilinear form that is defined for variational strain energy in an elastic continuum problem as an explicit form of the coercive bilinear form. The governing equation of the velocity indicates that we can determine the velocity as a displacement of the pseudo-elastic body defined in the design domain by loading a pseudo-external force in proportion to the negative value of the shape gradient function under constraints on the displacement of the invariable boundaries. We call this solution the traction method because of this procedure. To conduct a numerical analysis, we can use any technique applicable to linear elastic problems, such as the finite element method or boundary element method.

This paper briefly describes the derivation of the shape gradient functions and the procedure of the traction method and presents numerical results for linear elastic continuum problems[11−15] and flow field problems[16−].[18] It also describes a method for coupling the traction method with the topological optimization method using micro-scale voids proposed by Bendsøe and Kikuchi,[19] and presents an application to actual problems.

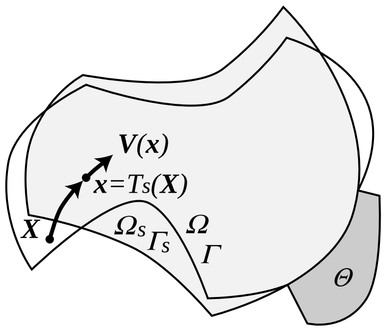


Figure 1 Domain variation

2 Domain Variation

Let an open set $\Omega \subset \Omega_{\text{limit}} \in \mathbb{R}^n$, $n = 2, 3$ be an initial domain with a boundary Γ and varied to a domain $\Omega_s \subset \Omega_{\text{limit}}$ with a boundary Γ_s. Assuming $\vec{T}_s(\vec{X})$, $s \geq 0$, $\vec{X} \in \bar{\Omega}$, as a one parameter family which is mapped from the initial closed domain $\bar{\Omega}$ to the varied closed domain $\bar{\Omega}_s$ as shown in Fig. 1, i.e. $\vec{T}_s(\vec{X}) : \bar{\Omega} \ni \vec{X} \mapsto \vec{x} \in \bar{\Omega}_s$, we can rewrite it by the ordinary differential equation with respect to s:

$$\vec{T}_0(\Omega) = \Omega, \quad \dot{\vec{T}}_s(\Omega) = \vec{V}(\Omega_s) \in D, \quad s > 0 \tag{1}$$

In this paper, for n dimensional vectors the notation $(\vec{\cdot})$ and the tensor notation with subscripts are used. Considering s as a time history, we call $\vec{V}$ the velocity function or velocity field. Assuming a restriction of domain variation on the subdomain or subboundary Θ of the design domain, the kinematically admissible set of the velocity field D is given by

$$D = \{\vec{V} \in \left(C^{0,\alpha}(\bar{\Omega}_s)\right)^n \mid \alpha > 0,\ \vec{V}(\vec{x}) = \vec{0},\ \vec{x} \in \Theta\} \tag{2}$$

where the Lipschitz condition $\vec{V} \in \left(C^{0,\alpha}(\bar{\Omega}_s)\right)^n$, $\alpha > 0$, is defined by $\exists C$, $|\vec{V}(\vec{x}_1) - \vec{V}(\vec{x}_2)| \leq C\|\vec{x}_1 - \vec{x}_2\|^\alpha$, $\forall \vec{x}_1, \vec{x}_2 \in \bar{\Omega}_s$.[5] For the sake of simplicity, we assume the measure of Θ is not zero.

When a domain functional J_{Ω_s} and a boundary functional J_{Γ_s} of a distributed function ϕ_s,

$$J_{\Omega_s} = \int_{\Omega_s} \phi_s \, dx \tag{3}$$

$$J_{\Gamma_s} = \int_{\Gamma_s} \phi_s \, d\Gamma \tag{4}$$

are considered, their derivatives with respect to s are given by

$$\dot{J}_{\Omega_s} = \int_{\Omega_s} \phi_s' \, dx + \int_{\Gamma_s} \phi_s \vec{n}^T \vec{V} \, d\Gamma \tag{5}$$

$$\dot{J}_{\Gamma_s} = \int_{\Gamma_s} \{\phi_s' + (\phi_{s,i} n_i + \phi_s \kappa) \vec{n}^T \vec{V}\} \, d\Gamma \tag{6}$$

where $\vec{n}$ is an outward unit normal vector and $(\,\cdot\,)^T$ denotes the transpose. The notation κ denotes the mean curvature. The shape derivative ϕ_s' of the distributed function ϕ_s indicates the derivatives under a spatially fixed condition. In the tensor notation, the Einstein summation convention and the gradient notation $(\,\cdot\,)_{,i} = \partial(\,\cdot\,)/\partial x_i$ are used.

3 Shape Gradient Function

Many basic domain optimization problems can be formulated in terms of the finding the domain $\Omega_s \subset \Omega_{\text{limit}}$ that minimize an objective functional $F(\vec{u})$ of a state variable function $\vec{u}$ subject to a variational form of a state equation $g(\vec{u}, \vec{w}) = 0$, $\forall \vec{w} \in W$, under a constraint on the measure of the domain:

$$\begin{aligned}
\text{Given} \quad & \Omega,\ M,\ \text{and coefficients in } g(\vec{u}, \vec{w}), \\
& \text{appropriately smooth} \\
& \text{and fixed in } \Omega_{\text{limit}}, && (7) \\
\text{find} \quad & \Omega_s = \vec{T}_s(\Omega), \\
& \dot{\vec{T}}_s(\Omega) = \vec{V}(\Omega_s) \in D, \quad s \geq 0 && (8) \\
\text{that minimize} \quad & F(\vec{u}), \quad \vec{u} \in U && (9) \\
\text{subject to} \quad & g(\vec{u}, \vec{w}) = 0, \quad \forall \vec{w} \in W, && (10) \\
& \text{meas}(\Omega_s) = \int_{\Omega_s} dx \leq M && (11)
\end{aligned}$$

where U and W are the addmissible set of the state function $\vec{u}$ and the variational state function, or adjoint functon, $\vec{w}$ respectively.

By applying the Lagrange multiplier method, or adjoint method,[4] and the formulae of derivatives of functionals with resipect to domain variation of the velocity field[2],[9] the Lagrange functional $L(\vec{u}, \vec{w}, \Lambda, \vec{T}_s)$ and its derivative to s, $\dot{L}$, are obtained as follows,

$$L = F(\vec{u}) + g(\vec{u}, \vec{w}) + \Lambda(\text{meas}(\Omega_s) - M) \tag{12}$$

$$\begin{aligned}
\dot{L} = & \, g(\vec{u}, \vec{w}') + g(\vec{u}', \vec{w}) + \dot{\Lambda}(\text{meas}(\Omega_s) - M) \\
& + l_G(\vec{V})
\end{aligned} \tag{13}$$

where $\vec{w}$ and Λ are the Lagrange multipliers of the state equation and measure constraint respectively. When $\vec{u}$ is determined by the state equation:

$$g(\vec{u}, \vec{w}') = 0, \quad \forall \vec{w}' \in W \tag{14}$$

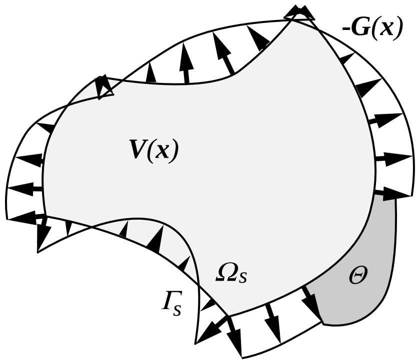


Figure 2 Traction method

$\vec{w}$ by the adjoint equation:

$$F(\vec{u}') + g(\vec{u}', \vec{w}) = 0, \quad \forall \vec{u}' \in U \tag{15}$$

and $\Lambda \geq 0$ by

$$\Lambda(\text{meas}(\Omega_s) - M) = 0 \tag{16}$$

$$\text{meas}(\Omega_s) \leq M \tag{17}$$

Eq. (13) becomes

$$\begin{aligned} \dot{L}|_{u,v,\Lambda} &= \dot{F}(\vec{u})|_{u,v,\Lambda} = l_G(\vec{V}) \\ &= \int_{\Gamma_s} \vec{G}_s^T \vec{V}\, dx = \int_{\Gamma_s} G_s \vec{n}^T \vec{V}\, dx \end{aligned} \tag{18}$$

The vector function $\vec{G}_s(\vec{u}, \vec{w}, \Lambda, \vec{T}_s)$ has the meaning of a sensitivity function to the velocity field $\vec{V}$ that we call the shape gradient function. The scalar function $G(\vec{u}, \vec{w}, \Lambda, \vec{T}_s)$ is called the shape gradient density function.

4 Traction Method

The traction method has been proposed as a procedure for solving the velocity field $\vec{V} \in D$ by

$$a(\vec{V}, \vec{w}) = -l_G(\vec{w}), \quad \forall \vec{w} \in D \tag{19}$$

where $a(\,\cdot\,,\,\cdot\,)$ is the bilinear form for the variational elastic strain energy defined by

$$a(\vec{u}, \vec{v}) = \int_{\Omega_s} C_{ijkl} u_{k,l} v_{i,j}\, dx. \tag{20}$$

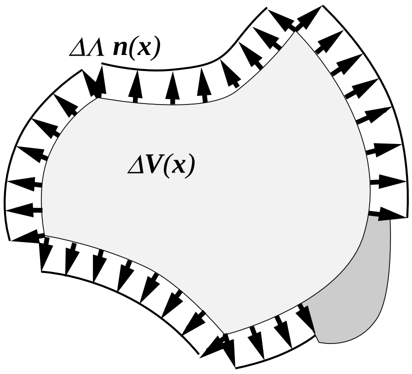


Figure 3 Domain measure control

where $l_G(\,\cdot\,)$ is the linear form defined by Eq. (18).

When the shape gradient density function G has appropriate smoothness and the measure of the restriction domain or boundary Θ is not zero, the velocity fields $\vec{V}$ can be determined by Eq. (19). That the solution $\vec{V}$ decrease the objective functional in convex problems is assured by using the coerciveness of the bilinear form $a(\,\cdot\,,\,\cdot\,)$.[9]

Equation (19) indicates that the velocity fields decreasing the objective functional are obtained as a displacement of the pseudo-elastic body defined in Ω_s by the loading of the pseudo-external force in proportion to $-\vec{G}_s$ under constraints on displacement of the invariable boundaries as shown in Fig. 2.

In this paper, FEM was employed to find the solution of Eq. (19).

The Lagrange multiplier Λ that satisfies Eqs. (16) and (17) is determined as follows. Since Λ contributes to the pseudo force $-\vec{G}_s$ as a uniform boundary force, the relation among the variation of the uniform boundary force $\Delta\Lambda\vec{n}$, the variation of the velocity $\Delta\vec{V}$ and the variation of the measure of the domain $\Delta\mathrm{meas}(\Omega_s)$ is obtained by elastic deformation analysis based on the following equation loaded with the uniform boundary force $\Delta\Lambda\vec{n}$ as shown in Fig. 3.

$$a(\Delta\vec{V},\vec{w}) = \Delta\Lambda\int_{\Gamma_s}\vec{n}^T\vec{w}\,d\Gamma, \quad \forall\vec{w}\in D \tag{21}$$

$$\Delta\mathrm{meas}(\Omega_s) = \int_{\Gamma_s}\vec{n}^T\Delta\vec{V}\,d\Gamma \tag{22}$$

The procedure of the traction method can be described as follows.

1. Start with a state function analysis followed by an adjoint function analysis, if necessary, depending on the problem to be solved.
2. Using the results, calculate the shape gradient function on the design boundary.

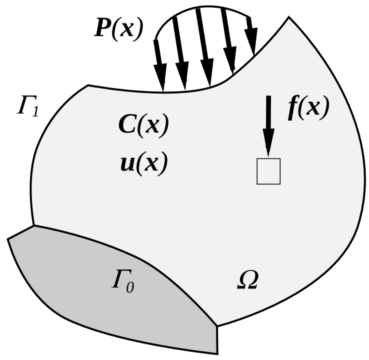


Figure 4 Domain variation problem of linear elastic continuum

3. Using the shape gradient function, analyze $\vec{V}$ by Eq. (19).

4. Deform the domain with $\vec{V}$ and evaluate the domain measure.

5. Determine Λ that satisfies Eqs. (16) and (17) using the results of Eqs. (21) and (22).

6. Multiplying the velocity function by an incremental value of, update the domain and return to step (1).

7. Terminate the procedure based on the results of the state function analysis.

5 Domain Optimization Problems

For some basic domain optimization problems in engineering, we can derive the shape gradient functions in the following manner.

5.1 Mean Compliance Minimization Problem

Let Ω be a domain of a linear elastic continuum loaded with a volume force $\vec{f}$ in Ω and a traction $\vec{P}$ on a boundary Γ_1 under a constraint on the displacement of a boundary Γ_0 as shown in Fig. 4.

A simple minimization problem of mean compliance by domain variation on a boundary $\Gamma_{\text{design}} \subset \Gamma_s$ under a constraint on the volume of the domain is formulated as follows.

$$
\begin{aligned}
\text{Given} \quad & \Omega,\ M,\ \vec{P},\ \vec{f}, \\
& \text{and } C_{ijkl},\ i,j,k,l = 1,2,\cdots,n, \\
& \text{appropriately smooth} \\
& \text{and fixed in } \Omega_{\text{limit}}, && (23) \\
\text{find} \quad & \Omega_s = \vec{T}_s(\Omega), \\
& \dot{\vec{T}}_s(\Omega) = \vec{V}(\Omega_s) \in D, \quad s \geq 0 && (24) \\
\text{that minimize} \quad & l(\vec{u}), \quad \vec{u} \in U && (25) \\
\text{subject to} \quad & a(\vec{u},\, \vec{w}) = l(\vec{w}), \quad \forall \vec{w} \in U && (26) \\
& \text{meas}(\Omega_s) = \int_{\Omega_s} dx \leq M && (27)
\end{aligned}
$$

where the bilinear form $a(\,\cdot\,,\,\cdot\,)$ is defined by Eq. (20), the linear form $l(\,\cdot\,)$ is defined by

$$
l(\vec{w}) = \int_{\Omega_s} f_i w_i \, dx + \int_{\Gamma_1} P_i w_i d\Gamma \tag{28}
$$

and U is the set of the admissible displacements satisfying $\vec{w} = \vec{0}$ on Γ_0.

For this problem, the shape gradient function is derived by using the Lagrange multiplier method and the formulae in Eqs. (5) and (6) as follows.[11]

$$
\begin{aligned}
G_s = & -C_{ijkl} u_{k,l} u_{i,j} + 2 f_i u_i \\
& +2(P_{i,j} u_i n_j + P_i u_{i,j} n_j + P_i u_i \kappa) + \Lambda
\end{aligned} \tag{29}
$$

5.2 Moving Problem of Vibrational Eigenvalues

Let us consider a moving problem of vibrational eigen-values $\lambda_{(r_m)}$, $m = 1, 2, \cdots, N$, of modal numbers r_m, $m = 1, 2, \cdots, N$, with eigen-modes $\vec{u}_{(r_m)}$,

$m = 1, 2, \cdots, N$, to a specified direction, or weights, $\alpha_{(r_m)}$, $m = 1, 2, \cdots, N$:

$$\begin{aligned}
\text{Given} \quad & \mathit{\Omega},\ M, r_m \text{ and } \alpha_{(r_m)},\ m = 1, 2, \cdots, N, \\
& C_{ijkl},\ i, j, k, l = 1, 2, \cdots, n,\ \text{density } \rho, \\
& \text{appropriately smooth} \\
& \text{and fixed in } \mathit{\Omega}_{\text{limit}}, && (30) \\
\text{find} \quad & \mathit{\Omega}_s = \vec{T}_s(\mathit{\Omega}), \\
& \dot{\vec{T}}_s(\mathit{\Omega}) = \vec{V}(\mathit{\Omega}_s) \in D, \quad s \geq 0 && (31) \\
\text{that maximize} \quad & \sum_{m=1}^{N} \alpha_{(r_m)} \lambda_{(r_m)} && (32) \\
\text{subject to} \quad & a(\vec{u}_{(r_m)}, \vec{w}) = \lambda_{(r_m)} b(\vec{u}_{(r_m)}, \vec{w}), \\
& \vec{u}_{(r_m)} \in U\ \forall \vec{w} \in U,\ m = 1, 2, \cdots, N, && (33) \\
& \text{meas}(\mathit{\Omega}_s) = \int_{\mathit{\Omega}_s} dx \leq M && (34)
\end{aligned}$$

where the bilinear form $b(\,\cdot\,,\,\cdot\,)$ is defined by

$$b(\vec{u}, \vec{w}) = \int_{\mathit{\Omega}_s} \rho u_i w_i \, dx. \tag{35}$$

The shape gradient function for this problem is derived as follows.

$$\begin{aligned}
G_s = \sum_{m=1}^{N} \alpha_{(r_m)} (&-C_{ijkl} u_{(r_m)k,l} u_{(r_m)i,j} \\
&+ \lambda_{(r_m)} \rho u_{(r_m)i} u_{(r_m)i}) + \mathit{\Lambda}
\end{aligned} \tag{36}$$

5.3 Frequency Response Minimization Problems

By putting three frequency response functionals of strain energy, kinetic energy and absolute mean compliance on the objective functionals, the following optimization problems are formulated.

$$\begin{aligned}
\text{Given} \quad & \mathit{\Omega},\ M,\ C_{ijkl},\ i,j,k,l=1,2,\cdots,n,\ \rho, \\
& \text{volume force } \vec{f}\cos\omega t, \\
& \text{and traction force } \vec{P}\cos\omega t, \\
& \text{appropriately smooth} \\
& \text{and fixed in } \mathit{\Omega}_{\text{limit}}, && (37) \\
\text{find} \quad & \mathit{\Omega}_s = \vec{T}_s(\mathit{\Omega}), \\
& \dot{\vec{T}}_s(\mathit{\Omega}) = \vec{V}(\mathit{\Omega}_s) \in D, \quad s \geq 0 && (38) \\
\text{that minimize} \quad & \frac{1}{2}a(\vec{u},\vec{u}),\ \frac{1}{2}b(\vec{u},\vec{u}) \text{ or } |l(\vec{u})|, \\
& \quad \vec{u} \in U && (39) \\
\text{subject to} \quad & -\omega^2 b(\vec{u},\vec{w}) + a(\vec{u},\vec{w}) = l(\vec{w}), \\
& \quad \forall \vec{w} \in U && (40) \\
& \text{meas}(\mathit{\Omega}_s) = \int_{\mathit{\Omega}_s} dx \leq M && (41)
\end{aligned}$$

5.3.1 Strain Energy Minimization Problem

The shape gradient function for the strain energy minimization problem is derived as

$$\begin{aligned} G_s = & \frac{1}{2}C_{ijkl}u_{k,l}u_{i,j} - C_{ijkl}u_{k,l}w_{i,j} \\ & +\omega^2 \rho u_i w_i + \mathit{\Lambda} \end{aligned} \tag{42}$$

where the displacement amplitude $\vec{u}$ and the adjoint displacement amplitude $\vec{w}$ are calculated with the modal displacement $\xi_{(m)}$ and the modal adjoint displacement $\eta_{(m)}$ by

$$\vec{u} = \sum_{m=1}^{\infty} \xi_{(m)}\vec{u}_{(m)}, \qquad \xi_{(m)} = \frac{l(\vec{u}_{(m)})}{\lambda_{(m)} - \omega^2} \tag{43}$$

$$\vec{w} = \sum_{m=1}^{\infty} \eta_{(m)}\vec{u}_{(m)}, \qquad \eta_{(m)} = \frac{\lambda_{(m)} l(\vec{u}_{(m)})}{(\lambda_{(m)} - \omega^2)^2}. \tag{44}$$

5.3.2 Kinetic Energy Minimization Problem

For the kinetic energy minimization problem, the shape gradient function is derived as:

$$G_s = \frac{1}{2}\omega^2 \rho u_i u_i - C_{ijkl}u_{k,l}w_{i,j} + \omega^2 \rho u_i w_i + \mathit{\Lambda} \tag{45}$$

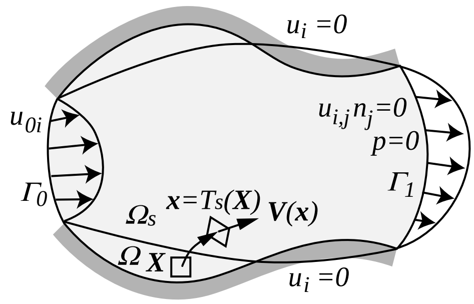


Figure 5 Domain variation problem in viscous flow field

where $\vec{u}$ is are calculated by Eq. (43) and $\vec{w}$ is calculated by

$$\vec{w} = \sum_{m=1}^{\infty} \eta_{(m)} \vec{u}_{(m)}, \qquad \eta_{(m)} = \frac{\omega^2 l_P(\vec{u}_{(m)})}{(\lambda_{(m)} - \omega^2)^2} \tag{46}$$

5.3.3 Absolute Mean Compliance Minimization Problem

For the absolute mean compliance minimization problem, the shape gradient function is derived as:

$$G_s = -C_{ijkl} u_{k,l} w_{i,j} + \omega^2 \rho u_i w_i + \Lambda \tag{47}$$

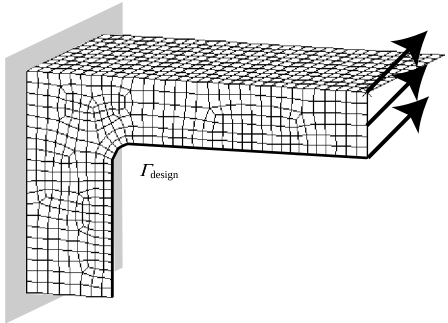
where $\vec{u}$ is are calculated by Eq. (43) and $\vec{w}$ is calculated by

$$\vec{w} = \begin{cases} \vec{u} & (l(\vec{u}) \geq 0) \\ -\vec{u} & (l(\vec{u}) < 0) \end{cases} \tag{48}$$

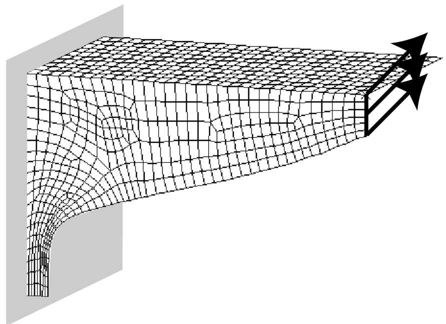
5.4 Dissipation Energy Minimization Problem of Viscous Flow Field

Let Ω be a flow field of an incompressible Newtonian fluid in a steady state as shown in Fig. 5. The fluid flows in from a boundary Γ_0 and flows out from a boundary Γ_1.

The minimization problem of dissipation energy by domain variation that occurs on a boundary $\Gamma_{\text{design}} \subset \Gamma$, $\Gamma_{\text{design}} \cap \Gamma_0 = \emptyset$, $\Gamma_{\text{design}} \cap \Gamma_1 = \emptyset$



Initial shape



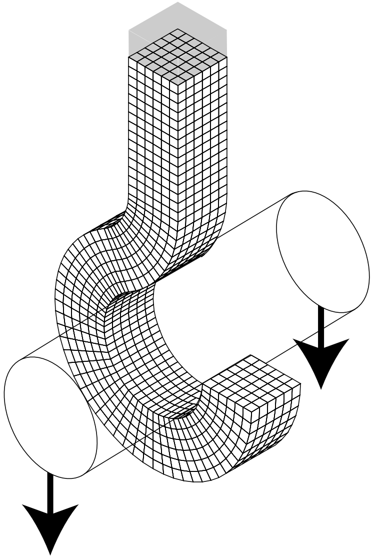
Minimized shape of mean compliance

Figure 6 Shell bracket clamped at left end

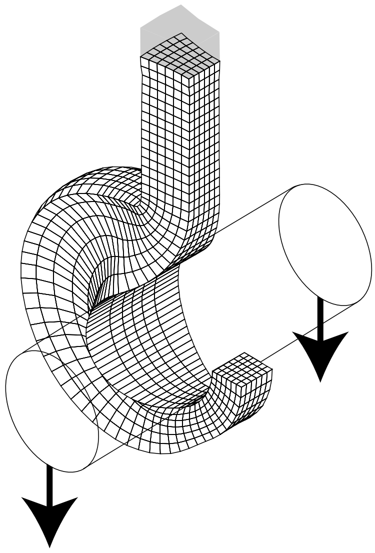
under a constraint on the volume of the domain is formulated as follows.

$$
\begin{aligned}
\text{Given} \quad & \Omega,\ M,\ \rho \text{ and viscous coefficient } \mu \\
& \text{appropriately smooth} \\
& \text{and fixed in } \Omega_{\text{limit}}, && (49) \\
\text{find} \quad & \Omega_s = \vec{T}_s(\Omega), \\
& \dot{\vec{T}}_s(\Omega) = \vec{V}(\Omega_s) \in D, \quad s \geq 0 && (50) \\
\text{that minimize} \quad & a(\vec{u},\vec{u}) + \bar{a}(\vec{u},\vec{u}), \quad \vec{u} \in U && (51) \\
\text{subject to} \quad & b(\vec{\nabla}\vec{u}^T\,\vec{u},\,\vec{w}) + a(\vec{u},\vec{w}) = \langle p, w_{i,i} \rangle, \\
& \quad \forall \vec{w} \in W, && (52) \\
& \langle q, u_{i,i} \rangle = 0, \quad \forall q \in Q && (53) \\
& \text{meas}(\Omega_s) = \int_{\Omega_s} dx \leq M && (54)
\end{aligned}
$$

where Eqs. (52) and (53) are variational forms of the Navier-Stokes equation and the continuity equation. The bilinear form $b(\,\cdot\,,\,\cdot\,)$ in the convective



Initial shape



Minimized shape of mean compliance

Figure 7 Hook clamped at top end

term is defined by Eq. (35) where $\vec{\nabla}\vec{u}^T\,\vec{u}$ is given as $u_{i,j}u_j$ in the tensor notation. The bilinear form $a(\,\cdot\,,\,\cdot\,)$ in the viscous term and $\bar{a}(\,\cdot\,,\,\cdot\,)$ in the dissipation energy are defined by Eq. (20) where $C_{ijkl} = \mu\delta_{ik}\delta_{jl}$ in $a(\,\cdot\,,\,\cdot\,)$ and $C_{ijkl} = \mu\delta_{il}\delta_{jk}$ in $\bar{a}(\,\cdot\,,\,\cdot\,)$. The bilinear form $\langle\,\cdot\,,\,\cdot\,\rangle$ in the pressure term

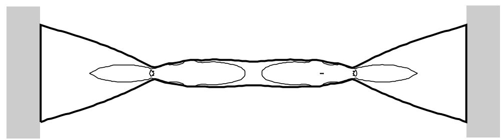
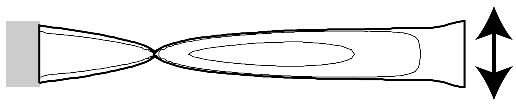
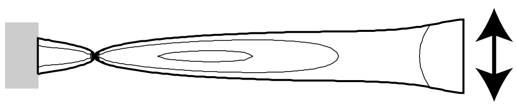


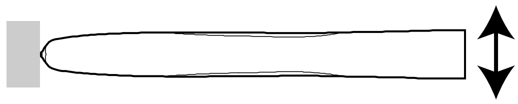
Figure 8 Maximized shape of $\lambda_1 + \lambda_2$ for a beam-like continuum clamped at both ends



Minimized shape of strain energy



Minimized shape of kinetic energy



Minimized shape of absolute mean compliance

Figure 9 Problem of beam-like continuum clamped at left end with non-structural mass of 10 % of structural mass at right end and excited between 1st and 2nd natural frequencies

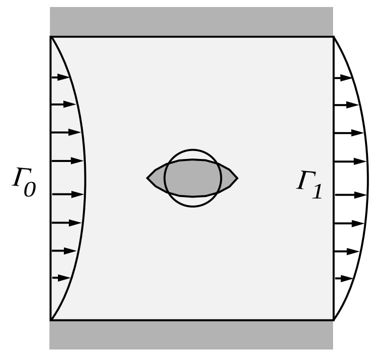


Figure 10 Minimized shape of dissipation energy for isolated body in channel

is defined as

$$\langle p, q \rangle = \int_{\Omega_s} pq \, dx \tag{55}$$

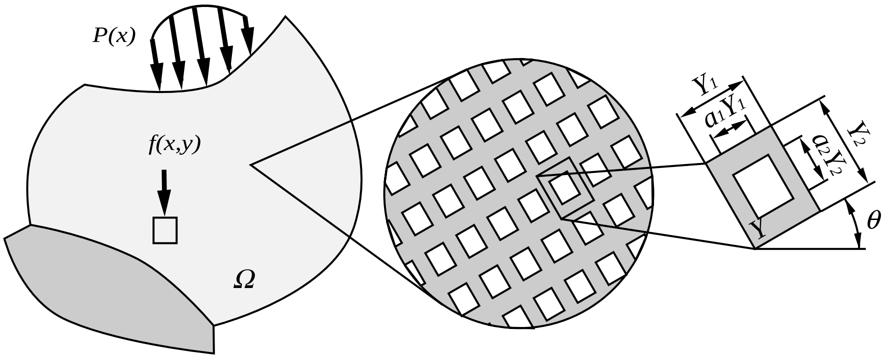


Figure 11 Topological optimization method using micro-scale voids

The velocity $\vec{u}$, the adjoint velocity $\vec{w}$, the pressure p and the adjoint pressure q are in the function spaces having appropriate smoothness, respectively,

$$U = \{\vec{u} \mid \vec{u} = \vec{u}_0 \text{ given on } \Gamma_0,\ u_{i,j}n_j = 0 \text{ on } \Gamma_1,$$
$$\vec{u} = \vec{0} \text{ on } \Gamma_s \setminus \Gamma_0 \cup \Gamma_1\} \tag{56}$$
$$W = \{\vec{w} \mid \vec{w} = \vec{0} \text{ on } \Gamma \setminus \Gamma_1\} \tag{57}$$
$$Q = \{p \mid p = 0 \text{ on } \Gamma_1\}. \tag{58}$$

Applying the Lagrange multiplier method, the shape gradient function for this problem is derived as follows.

$$G = -\mu w_{i,j}u_{i,j} + \mu u_{i,j}(u_{i,j} + u_{j,i}) + \Lambda \tag{59}$$

where the adjoint velocity $\vec{w}$ are calculated by

$$b(\vec{\nabla}\vec{u}'^T\,\vec{u},\,\vec{w}) + b(\vec{\nabla}\vec{u}^T\,\vec{u}',\,\vec{w}) + a(\vec{u}',\vec{w})$$
$$-2\{a(\vec{u}',\vec{u}) + \bar{a}(\vec{u}',\vec{u})\} = \langle q,\,u'_{i,i}\rangle, \quad \forall\vec{u}' \in U \tag{60}$$
$$\langle p',\,w_{i,i}\rangle = 0, \quad \forall p' \in Q \tag{61}$$

6 Numerical Results

For the optimization problems provided with shape gradient functions, we can apply the traction method.

Figure 6 and 7 show optimized shapes for the mean compliance minimization problems obtained by a shape analysis system developed with a general purpose FEM code.[12]

The result for the moving problem of vibrational eigenvalues is shown in Fig. 8.[14] This problem involved finding the maximized shape of. For

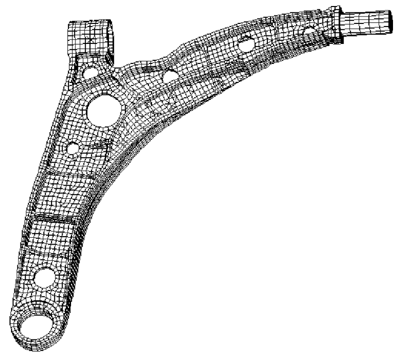


Figure 12 Original design of an automotive suspension system part

the frequency response minimization problems, we obtained various results depending on the objective functional as shown in Fig. 9.[15] We confirmed, however, that the objective functional decreased monotonously with increasing iteration.

For the dissipation energy minimization problem of a channel with an isolated body assuming the Stokes flow, the optimized shape is shown in Fig. 10.[16]

7 Coupling With Topological Optimization Method Using Micro-Scale Voids

An attempt was also made to couple the traction method with the topological optimization method using micro-scale voids as proposed by Bendsøe and Kikuchi.[19] Their approach is based on the idea of periodic microstructures as shown in Fig. 11. They formulated a domain optimization problem, including a consideration of topology, to find the size and orientation functions of microstructures. They found the relationship between the design functions and macro properties of the material by using the theory of the homogenization method.

Figures 12 and 13 shows the process optimized for one part of an automotive suspension system with the objective of minimizing the mean compliance analyzed by a topology and shape analysis system developed with a general purpose FEM code. Figure 12 shows the original design of this automotive part. We started with a two-dimensional problem. Assuming the design domain as shown in Fig. 13, we obtained the optimized topology by the topological optimization method. Based on the result, we made a three-dimensional model by selecting elements filled with the material. Applying the traction method to the three-dimensional model, we finally obtained the shape shown in Fig. 13.

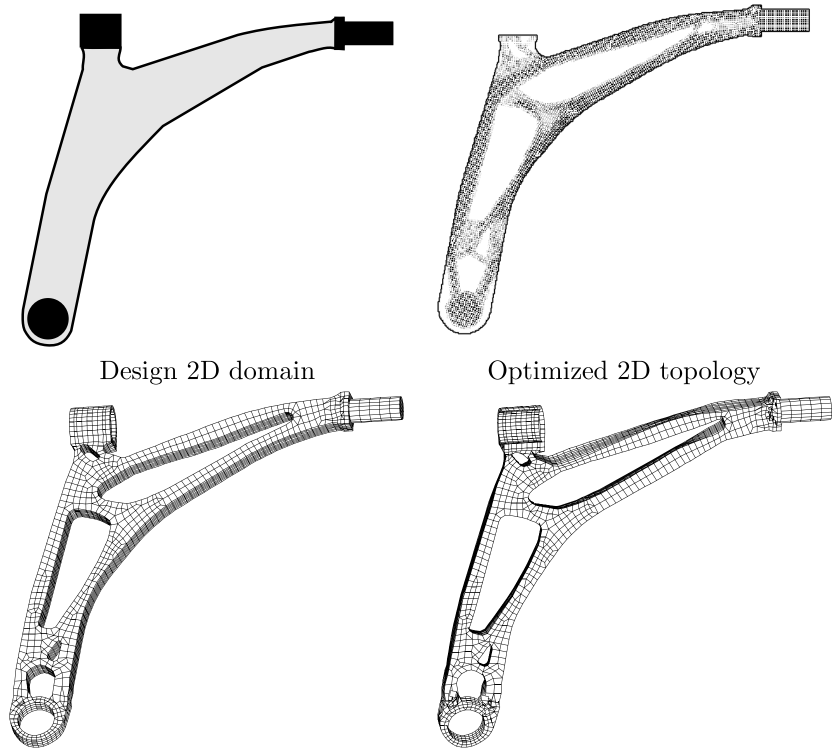


Figure 13 Process Optimized for One Part of Automotive Suspension System with Objective of Minimizing Mean Compliance

8 Conclusion

This paper has presented a numerical analysis technique for application to shape optimization problems of linear elastic continua and flow fields. The technique can be implemented using general purpose FEM codes. By coupling this technique with the topological optimization method, a more sophisticated system can be developed.

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